Why Can't I Divide by Zero?

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This is a fair question and one that should be given a response using the mathematical principles that govern all our arithmetic. So, here goes.

Let's start first with a principle that governs arithmetic operations. It has two parts:

1. The operation must have a result
2. The result must be unique. That is, you can't have more than one answer.

Consider the following example:

$$10 \div 5 = 2$$

We can check our result of 2 by checking: 2 x 5 = 10. Yea! So we have met condition one. Now, are there other possible answers to $10 \div 5$ that work? Nope, try as we might, there is no other way to multiply 5 by another number than 2 and still get 10.

Next, let start by trying to divide 10 by 0: $10 \div 0 = ?$ What can we multiply by 0 and get 10? Let's try some: 5 x 0 = 0 (didn't get 10). Let's try again: 10 x 0 = 0. Oops. That didn't work either. In fact, no matter what we pick as a possible solution, we can't get the correct result we need. ? x 0 = 0 and never will be 10. In this case we have failed to fulfill condition 1. This will be true for any non-zero dividend we pick.

Oh, Oh, Oh. How about 0 ÷ 0? That will work. Watch: 0 x 0 = 0. There! We can divide by zero provided we are dividing into 0.

Uh, NOT SO FAST! I think 0 ÷ 0 = 62. "Not correct," you say. Let's check: 0 x 62 = 0. There, see? 62 works.

Hopefully, you are ready to answer back, "Well, not so fast." Maybe the answer to 0 ÷ 0 is 500? Sounds ridiculous. Let's check. 0 x 500 = 0. That works, so 500 is also an answer. How do we resolve this quandary?

Easy! Let's go back and look at the conditions governing arithmetic operations. The second one says the operation must have a **unique** result. 0 ÷ 0 has infinitely many results. 62 works, 500 works, ½ works. So does -4, and on and on. Pick any really number you want. It will work as an answer for 0 ÷ 0.

Thus, since division by 0 fails on both conditions, we cannot divide by 0.