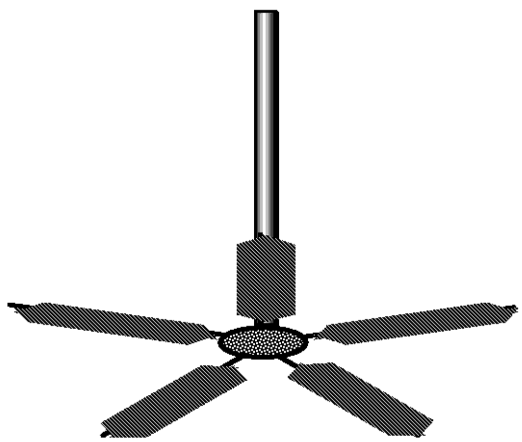
San Antonio Technology In Education Coalition

Exponential Functions

The Speed of a Fan



This lesson was developed under a grant funded by the United States Department of Education, Office of Education Research and Improvement.

A. Student Performance Objectives

(b) **Foundations for functions**: knowledge and skills and performance descriptions.

(1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.

(A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

(e) **Rational functions**: knowledge and skills and performance descriptions. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. Following are performance descriptions.

(2) The student analyzes various representations of rational functions with respect to problem situations.
(3) For given contexts, the student determines the reasonable domain and range values of rational functions, as well as interprets and determines the reasonableness of solutions to rational equations and inequalities.
(4) The student solves rational equations and inequalities using graphs, tables, and algebraic methods.
(5) The student analyzes a situation modeled by a rational function, formulates an equation or inequality composed of a linear or quadratic function, and solves the problem.

(f) **Exponential and logarithmic functions**: knowledge and skills and performance descriptions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. Following are performance descriptions.

(2) The student uses the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describes limitations on the domains and ranges, and examines asymptotic behavior.
(3) For given contexts, the student determines the reasonable domain and range values of exponential and logarithmic functions, as well as interprets and determines the reasonableness of solutions to exponential and logarithmic equations and inequalities.
(4) The student solves exponential and logarithmic equations and inequalities using graphs, tables, and algebraic methods.
(5) The student analyzes a situation modeled by an exponential function, formulates an equation or inequality, and solves the problem.

B. Critical Mathematics Explored in this Activity

In this activity, students will explore the Exponential function in the form of $f(x) = a + be^{cx}$. The students will also review the rational function in the form of $f(x) = a + \frac{b}{cx + d}$.

C. How Students will Encounter the Concept

The students are going to analyses data collected from a ceiling fan that is slowing down. The students will be comparing the time interval between blades for each one-fifth rotation of the fan. The students will write equations based upon this data and then make interpolations and extrapolations. The students will then be asked to convert the intervals of time to the speed of the fan and then examine this new data--with emphasis on the meaning of the x- and y-intercepts.

D. What the Teacher should do to Prepare

Students should each have access to a computer with Graphical Analysis, the file "**FAN'**, a graphing calculator, and a copy of the lesson.

Setting up

The teacher must load a copy of the file "**FAN"** on each computer and instruct the students as to how they are to access the file.

E. Assessment of Students During the Activity

The first set of data the students will use is the collection of intervals of time for each one-fifth rotation of the fan. Students will associate this with the speed at which the fan is spinning. The students must realize that the interval of time between any two blades and the speed vary inversely with one another.

Writing the general equation with the parameter changes in them may be a bit tricky for some students. The exponential function will be $f(x) = a + be^{cx}$ and the rational function will be $f(x) = a + \frac{b}{cx + d}$.

Note that in the exponential function, there is no "+d" parameter. It can be shown to the students that this is actually omitted through the laws of exponents.

In problem number 10, the students are asked to do an automatic curve fit using their general equation. Graphical Analysis may need some help with its initial values. Try a=.1, b=1, c=1. This could be a good time to discuss parameter changes and have students guess at these values.

F. Answers

1. The independent variable is time.
2. The dependent variable is the period.
3. As the fan slows down, the period increases.
4. Any increasing graph would be appropriate here.

5 If not similar, than look for an explanation as to why the two graphs are different.

6. There is a positive correlation because as the time increases, the period is also increasing.
7. Approximately 0.3 seconds
8. Exponential
9. $f(x) = a + be^{cx}$
10. $y = 0.305 + .0548e^{.0886x}$

    (Note, Graphical Analysis may need some help with its initial values, if so, use a=.2, b=1, c=1)

11. $y = 0.305 + .0548e^{.0886(8.2)}$
    $y = 0.3976\text{sec}$
12. $0.65 = 0.305 + .0548e^{.0886(x)}$
    $x = 31.3968\text{sec}$
13. As the time continues, the period appears to keep getting larger (unbounded).

14 No because the fan eventually stops in reality and therefore there would be no period to calculate.

15. Distance= $\frac{1}{5}(2\pi(2)) = 2.51327$
16. Rational
17. a negative correlation
18. As the time increases the speed of the fan decreases.

19. $f(x) = a + \dfrac{b}{cx + d}$

20. $y = -6.17 + \dfrac{6.07}{.00694x + .405}$

21. $\dfrac{1}{2} = -6.17 + \dfrac{6.07}{.00694x + .405}$

$x = 72.8\text{sec}$

22. Yes the graph has a y-intercept. It is the speed of the fan when it is first turned off.

23 8.817ft/sec

24 Yes the graph has an x-intercept. It is the time it takes the fan to stop spinning.

25. 83.399sec.

F. Homework

Speed of two Fans--Reflect and Apply

Answers

1. At time 0, the first fan has a period of .349 sec and the second fan had a period of .350 sec. Therefore, since the period of the first fan is slightly less than that of the second, the speed of the first fan must be greater than the speed of the second fan.

2.

$$\begin{aligned} y &= .309 + .04e^{.077(12)} \\ y &= .309 + .04e^{.924} \\ y &= .309 + .04(2.5193) \\ y &= .309 + .10077 \\ y &= .40977 \end{aligned} \qquad \begin{aligned} y &= .314 + .036e^{.071(12)} \\ y &= .314 + .036e^{.852} \\ y &= .314 + .036(2.3443) \\ y &= .314 + .0844 \\ y &= .3984 \end{aligned}$$

3. By graphing the two equations, the first fan's graph increases faster than the second fan's graph. This implies the first fan has a higher period, and thus a longer time interval, than the second fan at any give time. Since the period is greater, the first fan's speed is slower than the second fan.

4. The distance traveled between any two blades on the fan is $d = \frac{1}{4}\left(2\pi(1.5)\right) = \frac{3\pi}{4} feet$. Dividing this distance by the period at 12 seconds (from #2) we get: 5.75 ft/sec.

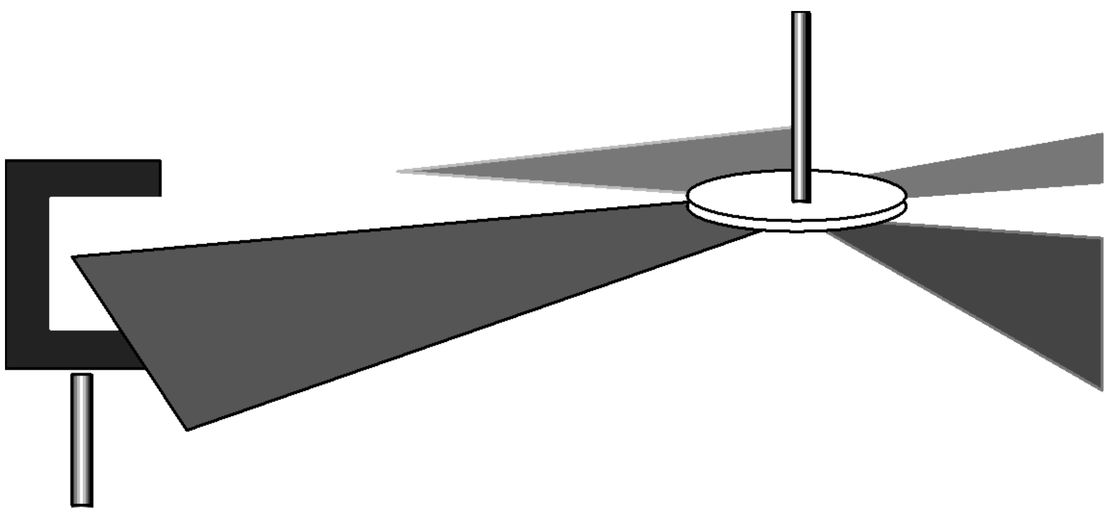
5. The distance traveled between any two blades on the fan is $d = \frac{1}{3}\left(2\pi(2)\right) = \frac{4\pi}{3} feet$. Dividing this distance by the period at 12 seconds (from #2) we get: 10.514 ft/sec.

G. Extensions

Name:______________________________Date:_____________Period:___

The Speed of a Fan

The speed of a fan, when it is first turned off, slows down. Veronica and Rodney want to find a function which models this event. To do so they set a Photo Gate at the precise height so that the blades of the fan would pass through the Photo Gate, triggering the sensor. They used the Pendulum file from Logger Pro to collect their data. Since there are five blades evenly spaced around the fan, this file will record the time interval between the fan blades for each 1/5 rotation of the fan. There are five blades evenly spaced around the fan. The setup Rodney and Veronica used is shown here.

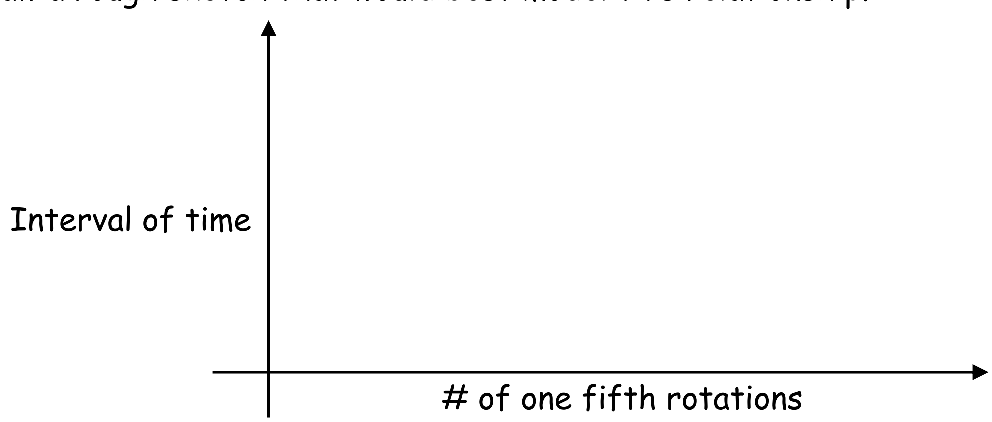


1. What is the Independent variable: ___________________________

2. What is the Dependent variable: ____________________________

3. What is happening to the time interval between the blades as the fan slows?
____

4. Draw a rough sketch that would best model this relationship.



After running the experiment in Logger Pro, and then transferring the data into Graphical analysis, Rodney and Veronica saved their data in a file called FAN. Open this file now.

5. How does this data, and the graph of the data points, compare to what you sketched in # 4.

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6. Why does this graph have a positive correlation?

____

____

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7. Can you tell, from the data, approximately what is the interval of time for the fan is when it is turned on?

____

8. What kind of function could this be? ______________________________

When doing an automatic curve fit, **it is best to type in your own function**, which takes into consideration the parameter changes that all functions have. The general form for any function is $y = a + b \cdot f(cx + d)$. For example, if you think this reltionship is the square root function you would type in $y = a + b\sqrt{cx + d}$, or for the quadratic function you would type in $y = a + b(cx + d)^2$. The letters a, b, c, and d represent numbers that cause parameter changes to the parent function.

9. What is the general form of the function you chose as your answer in #8. Write that function here.

____

10. Use the automatic curve fit of Graphical Analysis and try your function. If your choice needs to be revised, do so now and try your new function. Continue to adjust your function until you get one that is a good fit for this data.
Write that equation here_______________________________________

11. Use your equation to find the time interval of the 32^{nd} fifth revolution from when the fan was turned off. ___________________________________

12. Use the equation to find when the fan had an interval of 0.650 sec.

____

13. What appears to be happening to the intervals of time as the fan spins down?

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14. Would this happen in reality? Why or why not?

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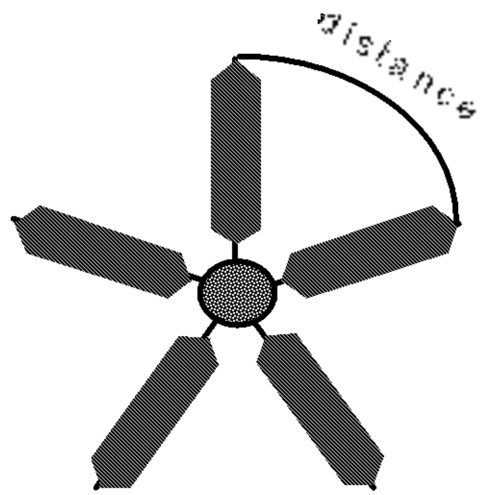
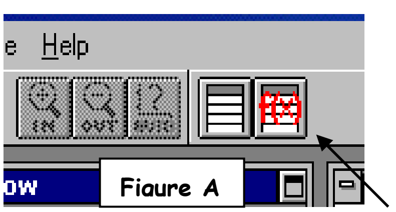
If the radius of the fan is 2 feet, and there are five blades evenly spaced around the fan (as shown to the right), calculate the distance from the center of one blade to the center of the other blade. (Hint the distance is a fraction of the circumference.)



15. Distance =_________________

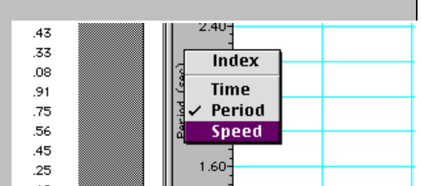
16. The speed of the fan is calculated by dividing the distance by the period. Since the distance between the blades is constant, what kind of function will we obtain for speed? _______________________



In Graphical Analysis, with the Data Table Window Activated, create a **"New column" that is <u>calculated</u>**. You may do this by pushing the new function button located on the top tool bar (Figure A).

In this **calculated** column place the formula
Speed = "[*your value for distance*] / Period".

The column will fill with values for the speed of the fan.



Select the speed values for the y-axis by clicking on the label "period" along the y-axis and choose speed.

17. What kind of correlation does this new graph have? ___________________

18. Explain why it has this correlation.

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19. Write the general equation (as you did in #9) that will models this new data:

____

20. Enter your function into Graphical Analysis and use manual curve fit to find the equation which will best model this data.

Write your equation here: ______________________________

21. Use your equation to find the time when the fan is spinning at 1/2 a foot per second.

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22. Does this graph have a y-intercept? If so, what is its real world meaning?

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23. Use the equation to find the y-intercept. __________________________

24. Does this graph have an x-intercept? If so, what is its real world meaning?

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25. Use the equation to find the x-intercept. __________________________